

IV. *Of the Moon's Distance and Parallax:*
A Letter to Andrew Reid, Esq; from P.
Murdoch, D. D. and F. R. S. 12 Nov.
 1763.

S I R,

Read Jan. 26, 1764. **I** Have at your desire wrote out what I was mentioning to you in our last conversation; of an easy rule for determining the Moon's distance, from the received theory of central forces: which I wish may merit your approbation: it will at least serve as a testimony of the esteem and regard with which I am, &c.

I.

Sir Isaac Newton investigated the law of gravitation, in the duplicate ratio of the distance of the central body inversely, from the following *data*.

1. The length of a simple pendulum which vibrates in one *second* of time, gave him, by Huygens' theorem, a determinate measure of the force of gravity, at the place of observation. And, by his own theory, he could thence infer the like measure for any other place, of a given latitude*.

2. The Earth's semidiameter was computed from the Abbé Picard's measure of a degree of the terrestrial meridian.

* See one of the Essays prefixed to *Busching's Geography*.

3. The

3. The Moon's parallax, as determined by the most skilful astronomers, gave him the Moon's distance in semidiameters of the Earth.

4. The time of a periodical month gave him the ratio of the versed sine of the arc of the Moon's orbit which she describes in one *second*, to the radius.

And from these his conclusion was; that the gravitation at the Earth's surface, being diminished as the square of the distance from the Earth's centre increases, would, at the distance of the Moon, produce a fall from rest, in one *second*, precisely equal to that versed sine. Or, that the gravitation of the Moon toward the Earth, being increased as the square of that distance is diminished, would, at the Earth's surface, be of the same quantity as that of falling bodies is (by the experiment of the pendulum) actually found to be.

II.

But the law of gravitation, thus deduced, being found to hold universally, and reciprocally, amongst all the great bodies of our system, so that even the minute anomalies of their motions are explained from it; we may now assume *it* as given, and make the *Moon's distance* the *quantity sought*.

Thus, writing F for the number of feet which a body falling from rest, describes, *in vacuo*, at the equator, in one *second*, V for the versed sine of the arc of the moon's orbit described in the same time, to the radius unity, D for the semidiameter of the equator in feet, and the ratio of the distance of the centers of the earth and Moon, to the semidiameter of

of the earth, that of X to 1: We shall have, by the general law, the Moon's fall in 1'', equal to $\frac{F}{X^2}$; but the same fall is equal to $V \times D \times X$; whence $X^3 = \frac{F}{V \times D}$, and $X = \sqrt[3]{\frac{F}{V \times D}}$ is the distance sought, in semidiameters of the equator.

Now a simple pendulum which beats seconds, measuring, at London, 39.126 inches; if the usual allowance is made for the weight of the air, and for the *Newtonian* figure of the Earth *, the weight ($\frac{1}{2883}$) taken off by the centrifugal force being likewise restored, a *second-pendulum* at the equator would be 39.154 inches long. And, by Huygens' rule, half this length is to the initial fall in one *second*, in the duplicate ratio of the diameter of a circle to its circumference: that fall therefore, at the equator, and *in vacuo*, is 16.10185 feet; the logarithm of which number is $1.2068645 = \log. F$.

The *toises* in a degree of the equator, or, which is the same, in a degree of the meridian at *lat.* $54\frac{3}{4}$, being nearly 57200, the logarithm of the number of feet English in the semidiameter of the equator,

that is $\log. D$ will be nearly — — 7.3211900,

And the $\log.$ versed sine of the Moon's } — 12.5492882,
arc in 1'', being — — — }

Their Sum — — — — — 5.8704782

taken from $\log. F$, leaves + 5.3363863, a third of which is 1.7787954, the logarithm of $X = 60.08906$ semidiameters of the equator.

* See the *Effay* quoted above.

And the arithmetical complement of this last logarithm, which is -2.2212046 , is the log. tangent of the Moon's mean horizontal parallax at the equator; which therefore, is $57' 12'', 34$.

III.

Such would be the distance of the Earth's and Moon's centers, were the Earth immoveable: but it is somewhat increased by their revolution round their common centre of gravity.

Writing $x + 1$ for that distance, divided by the centre of gravity in the ratio of x to 1 ; imagine a sphere of the same dimensions as our earth, placed at that centre, to exert the same attractive force on the Moon as our Earth actually does, the periodic time remaining unaltered: then must the density of this sphere be diminished in the ratio of x^2 to $x + 1$, that its nearer distance from the Moon may be compensated by the defect of density and attractive force. If now an inhabitant of the fictitious earth were supposed to compute its distance from the Moon, in the manner just now shewn; the quantities V and D would be the same as in the former calculation; but his f would be to our F , as x^2 to $x + 1$, and thence, his x would be to our X as $x^{\frac{2}{3}}$ to $x + 1$, that is, $x = \frac{x}{x + 1}^{\frac{2}{3}} \times X$.

This is the distance from the fictitious Earth, or from the common centre of gravity; but (T) the distance from our Earth, is $\frac{x + 1}{x} \times \frac{x}{x + 1}^{\frac{2}{3}} \times X$, greater,

as was supposed, in the ratio of $x+1$ to x ; that is,

$$T = \left[\frac{x}{x+1} \right]^{-1} \times \left[\frac{x}{x+1} \right]^{-\frac{2}{3}} \times X = \left[\frac{x}{x+1} \right]^{-\frac{1}{3}} \times X = \sqrt[3]{\frac{x+1}{x}} \times X.$$

Sir Isaac Newton, from the *phenomena* of the tides, estimated the ratio of $x+1$ to x to be that of 40.788 to 39.788. In that case, the cubic root of $\frac{x+1}{x}$ will have for its logarithm 0.0035934; which added to 1.7787954, the logarithm of X computed for an immoveable earth, gives 1.7823888, the logarithm of 60.5883 semidiameters of the equator. And the Moon's horizontal parallax, for this distance, is 56' 44'',07.

IV.

On the other hand, if we had observations of the Moon's parallax (and distance) which could be reckoned exact enough for the purpose, we might thence determine the ratio of x to 1, that is, the ratio of the quantities of matter in the Earth and Moon.

For having $\frac{T}{X} = \sqrt[3]{\frac{x+1}{x}}$, and $\frac{T3}{X3} = \frac{x+1}{x}$; likewise T being given from observation, and X computed as above; it is manifest that the ratio of $x+1$ to x , and, by division, that of x to 1, or of the mass of the earth to that of the Moon, is given.

For example, if it should be concluded from good observations, that T, the Moon's mean distance, is $60\frac{1}{2}$ semidiameters of the equator; for the logarithm of this distance, which is 1.7817554, take the logarithm of X, or 1.7787954, thrice the remainder

will be 0.00888, the logarithm of $\frac{x+1}{x} = 1.02066$; and the masses of the Earth and Moon would, on this supposition, be as 48.4027 to 1.

In all this, a small variation from the law of attraction, arising from the spheroid-figure of the earth, is neglected as inconsiderable; which it will be found to be by whoever takes the trouble to compute its quantity and effects.

R E M A R K S.

1. If F and D were taken of their just quantities, the Moon's horizontal parallax for an immoveable Earth being, at the equator, $57' 12\frac{1}{3}''$, is a *limit* which the true mean parallax cannot exceed: and the correspondent distance 60.08906 is a *limit* which the distance cannot fall short of: both being computed upon the supposition that $x+1=x$, or that the matter of the Moon is as nothing in comparison of the Earth. Nor can the parallax and distance be supposed to lye very near these limits, without leaving too little attractive force in the Moon to raise the tides.

2. If the Moon's mean apparent semidiameter is $15' 38\frac{1}{4}''$, and the distance of the centers 60.5883 semidiameters of the equator, according to Sir Isaac's estimate of the masses; the semidiameter of the Moon will be 0.275601 parts of the semidiameter of the equator, or .2763 of a mean semidiameter of the Earth. And the magnitudes of the Moon and Earth being as the cubes of their semidiameters, if the inverse ratio of their magnitudes is joined to the

direct ratio of their masses (1 to 39.788) the sum will be the ratio of their densities, that of 1.19143 to 1, a little less than 6 to 5.

3. Supposing still the same semidiameter of the orbit as before, the force of gravity will be to the Earth's attractive force on the Moon as 3670.94 to 1, and to the Moon's force on the Earth as 40.788 times that number, or 149730.4, to 1.

Again, the force of the Moon upon that surface of the ocean to which she is vertical, being to her force on the Earth's centre, as the square of 60.5883 to that of 59.5883; and the difference of these squares being to the latter as 1 to 29.54623, this difference of the forces will support the weight of one 4423968th part of the water at the vertex. And, because the Earth's semidiameter is small in comparison to the Moon's distance, the like differences of force will decrease from the surface to the centre, nearly in an arithmetical progression, as the weight of the water does; making the case analogous to the diminution of gravity by centrifugal force.

But it is likewise easily shewn, that half this quantity of lunar force exerts itself to depress the waters all around at the distance of 90 degrees from the vertex; $\frac{3}{4}$ therefore of the former fraction, that is one 2949312 part of the force of gravity, will be the total cause of the difference in height of the *flood* and *ebb*, in an open and boundless ocean.

Say therefore, if (in determining the figure of the earth) $\frac{1}{2889}$ of gravity, suspended by the centrifugal force, gave, for the difference of diameters $\frac{1}{831}$, what will one 2949312 part give? and the answer, in feet, will be 8.887.

4. In like manner, if we take $8''\frac{3}{4}$ for the Sun's parallax, and, thence, his distance from the Earth 23468,6 semidiameters of the equator, we shall find that his *whole* force to produce a difference of *Flood* and *Ebb*, is to his force at the Earth's centre, as 1 to $7823\frac{2}{3}$. But the Sun's distance being to the radius of the Moon's orbit as 387.34535 to unity, this last force will be to that of the Earth on the Moon, as 387,34535 to 178.7234 (by cor. 2. prop. princip. I.) And the Earth's force on the Moon is to gravity as 1 to the square of 605883; whence, adding these ratios, the Sun's force to move the sea will be to the force of gravity as the fraction whose logarithm is — 8.1778026 to 1; or gravity is to that force as 13249445 to unity. And therefore, by the same analogy as above, we find the difference of *Flow* and *Ebb*, from the Sun alone, to be 1.97824; one foot $11\frac{3}{4}$ inches.

The solar force therefore, in raising the tides, is to the lunar, as 1 to 4.4924, in a ratio somewhat less than that computed by Sir *Isaac*. The ratio likewise of the sum of the forces to their difference is but 7.869 to 5, instead of 9 to 5, which he assumes from comparing the *spring* and *neap*-tides at *Bristol*. And it is indeed surprizing, how he could, from that *datum*, arrive at conclusions so near the truth, as his very probably are. He tells us he used the ratio of 9 to 5, only till a more certain could be procured, And therefore the foreign mathematicians, who have censured him on that head, and on some other articles of this doctrine, might have spared their reflexions; at least till they could shew that their own deductions were more agreeable to nature and observation.

6. Unity

6. Unity representing the force of gravity, d the Sun's distance, the Earth's force on the Sun will be $\frac{1}{d^2}$, or the fraction whose logarithm is $-9.259021.8$. And the solar force on the Earth is (from the numbers in remark 4) to the force of gravity as 1 to 1673.1: whence the attractive forces (and masses) of the Sun and Earth will be as 325172,3 to 1. Add to this the inverse ratio of their magnitudes, collected from the Sun's mean apparent semidiameter $16'.6''$, and the parallax $8''\frac{4}{5}$; and the density of the Sun will be to that of the Earth, as 1 to 4.068.

All this upon the supposition that the masses of the Earth and Moon are as 39.788 and 1. Hereafter, when the Moon's distance shall be more certainly known, that element may be corrected, and the operations repeated.

As to the Sun's parallax $8''\frac{4}{5}$, it cannot be much affected by any future determination of the Moon's distance. Nor is it here assumed of that quantity, at random; but from a theorem deduced from the established principles. I am, however, too diffident of myself to communicate it at present: because, altho' it agrees very well with Mr. *Short's* conclusion from the *Transit of Venus*, it differs considerably from that which a very learned and justly celebrated author hath lately published.

Note, The periods, as assumed in this paper, are; a *fidereal year* of 365.2563923 days; the periodical month 27.32165835 days.

I N D E X.

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T H E E N D O F V O L. L I V.

E R R A T A.

P. 33. l. 2. read thus: $T = \frac{x}{x+1}$ ⁻¹ $\times \frac{x}{x+1}$ ² $\times x$ &c.

14. read reckoned.

18. for $\frac{T^3}{\times 3}$ read $\frac{T^3}{\times^3}$

36. 11. from the bottom, to 8 put to 5.

99. ult. for greatly deformed, read somewhat deformed.